Market Risk Premium Expectation: Combining Option Theory with Traditional Predictors*

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Abstract

The market risk premium is central in finance, and has been analyzed by numerous studies in the time-series predictability literature and by growing studies in the options literature. In this paper, we provide a novel link between the two literatures. Theoretically, we derive a lower bound on the equity risk premium in terms of option prices and state variables. Empirically, we show that combining information from both options and investor sentiment significantly improves the out-of-sample predictability of the market risk premium versus using either type of information alone, and that adding an economic upper bound raises predictability further.

Keywords: Out-of-sample predictability, equity risk premium, index options, sentiment, recovery

JEL Classification: G1, G11, G12, G17

1 Introduction

The expected equity market excess return, or the market risk premium, is one of the central quantities in finance and macroeconomics. Going as far back as Dow (1920), the literature on time-series predictability attempts to shed light on what economic and financial variables drive the market risk premium. Fama and French (1988, 1989), Campbell and Shiller (1988a,b, 1998), and Huang, Jiang, Tu, and Zhou (2015), for example, find that dividend-price ratio, earnings-price ratio and investor sentiment can predict market returns. Rapach and Zhou (2022) provide a recent survey of the literature. Breaking new ground, Martin (2017) shows that option prices prove useful on the future market return, generating a substantial number of related research such as Kremens and Martin (2019); Martin and Wagner (2019); Kadan and Tang (2020); Chabi-Yo and Loudis (2020); Back, Crotty, and Kazempour (2022). However, the out-of-sample predictability uncovered by both strands of literature is still small.

In this paper, we provide the first study that combines two important lines of literature—time-series predictability and option recovery theory—to predict the market risk premium. We derive a new bound (a combined predictor) that incorporates the risk-neutral volatility computed from option prices and the traditional financial and macroeconomic state variables. We show that the new predictor performs well in out-of-sample forecasts and generates substantial economic gains consistently over time. In particular, it outperforms substantially than those when the method of each of the literature is used alone.

Theoretically, we follow Martin's (2017) procedure but without explicitly assuming the hypothesis of negative correlation condition (NCC). Instead, we incorporate the state variables into informative bounds on the market excess return expected in the future. In contrast to all existing extensions of Martin (2017), our study is the first to consider the role of state variables, making it possible to link the bounds to the broad classic time-series predictability literature that identifies various economic risks that impact the market.

Empirically, we construct the combined predictors using several sentiment indices in light of behavioral finance. We calculate the out-of-sample R^2 statistics (R^2_{OS}) relative to the historical market mean as a benchmark. We find that neither Martin's (2017) bounds nor sentiment variables deliver consistent out-of-sample outerperformance if used alone: the R^2_{OS} statistics are mostly negative or positive but insignificant based on Clark and West's (2007) tests. Moreover, we find that Back et al.'s (2022) 'bound + past mean slackness' strategy fails to improve the out-of-sample performance. In contrast, by combining the sentiment variables with the option bounds, we observe a substantial improvement in out-of-sample R^2_{OS} statistics. The R^2_{OS} statistics become mostly positive and are statistically significant. For instance, combing Martin's (2017) option bound with Rapach, Ringgenberg, and Zhou's (2016) short interest index generates a significant R^2_{OS} statistic of 0.695% compared with −1.450% for Martin's (2017) bound and −0.776% for short interest index. Moreover, we pool the individual forecasts from three combined predictors as Rapach, Strauss, and Zhou (2010) argue that pooling can better regularize forecast variability. For the evaluation periods between 2001 and 2020, pooling generates consistently significantly positive R^2_{OS} statistics of 1.417%, 6.065%, 14.105%, and 26.975%, for 1-month, 3-month, semi-annual, and annual return forecasts, respectively. Finally, our forecast encompassing and stabilization tests support that combining information from option prices and sentiment-based variables significantly improves the out-of-sample predictability relative to using either type of information alone.

We also consider imposing a simple upper bound on our forecasts. In the predictability literature, Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014) are the pioneering examples that incorporate economic constraints into the forecasts. Since the option theory essential provides lower bounds, we hence examine upper bounds only. For simplicity, based on MacKinlay (1995) and Cochrane and Saa-Requejo (2000), we impose an upper bound on the Sharpe ratio varying from 0.6 to 1. We find that such an upper bound helps improve the market risk premium predictability even further, with R^2_{OS} reaching 2.383% for 1-month return forecast.

The statistically superb performance by combing option theory with sentiment-based variables is also economically valuable. We show that the combined predictors, on average, generate higher average returns, greater out-of-sample Sharpe ratios, and larger certainty equivalent returns. Moreover, by computing Fleming, Kirby, and Ostdiek's (2001) performance fee, we find that a mean-variance investor would be willing to pay more than 300 basis points per year to switch from the historical mean to acquire the forecast from the combined predictors. For example, relative to the historical mean benchmark, the performance fees are around −442 and 189 basis points if Martin's (2017) option bound and Rapach et al.'s (2016) short interest index are used separately, but jump to 385 basis points if the two are combined.

Our paper contributes to the two important lines of literature on market risk premium. The first line, with Martin (2017) as a notable example, investigates how elusive it is to estimate the (conditional) expected return, which dates back to Merton (1980); Black (1993); Elton (1999). The topic is cutting-edge and invigorates a sequence of research complementarities, including Chabi-Yo and Loudis (2020) on the aggregate market; Martin and Wagner (2019); Kadan and Tang (2020); Chabi-Yo, Dim, and Vilkov (2022) on individual stocks; Clark, Lu, and Tian (2021) on equity forward return; Kremens and Martin (2019) on the foreign currency; Bakshi, Gao, and Xue (2022) on the treasury market; and Liu, Tang, and Zhou (2022) on the Federal Open Market Committee risk premium. The second line, as emphasized by Spiegel (2008) in *The Review of Financial Studies*, challenges researchers on whether our empirical model forecast the equity premium any better than the historical mean. The studies in this line focus on the out-of-sample predictability of the aggregate stock market return via extensions of the conventional predictive regression approach, including Neely, Rapach, Tu, and Zhou (2014) on technical indicators; Han, Lu, and Zhou (2022) on macro trends and economic priors; and Dong, Li, Rapach, and Zhou (2022) on long-short anomaly portfolios.

The remainder of this paper is organized as follows. Section 2 presents the theory. Section 3 discusses the empirical framework, followed by the out-of-sample test in Section 4. Section 5 provides a battery of robustness checks. Section 6 concludes.

2 Theory

In this section, based on Martin (2017), we provide the theoretical underpinnings for the important role of state variables. To facilitate our discussion, we employ the following notations, for a discrete time subscript t,

- S_T = price of a stock market index (inclusive of dividends) at future time T;
- $R_T \equiv \frac{S_T}{S_t}$ = gross market return over t to T. We assume that $R_T > 0$;
- $\mathbb{P}$ = the real-world probability measure, and the information set at time t is $\mathscr{F}_t$;
- $\mathbb{Q}$ = the risk-neutral probability measure;
- M_T = stochastic discount factor (SDF) with $\mathbb{E}_t^{\mathbb{P}}(M_T R_T) = 1$ holding;
- $R_{f,t} = \frac{1}{\mathbb{E}_t^{\mathbb{P}}(M_T)} = \mathbb{E}^{\mathbb{Q}}(R_T)$ = gross risk-free return over t to T (known at time t);
- $E_t^{\mathbb{P}}(x)$ = conditional expectation of a random variable under $\mathbb{P}$;
- $Cov_t^{\mathbb{P}}(x,y)$ = conditional covariance between two random variables under $\mathbb{P}$; and
- $Var_t^{\mathbb{Q}}(x,y)$ = conditional variance under $\mathbb{Q}$.

2.1 Negative correlation condition

Martin (2017) decomposes the market risk premium into two components

$$\begin{aligned}\mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t} &= \left[\mathbb{E}_t^{\mathbb{P}}(M_T R_T^2) - R_{f,t}\right] - \left[\mathbb{E}_t^{\mathbb{P}}(M_T R_T^2) - \mathbb{E}_t^{\mathbb{P}}(R_T)\right], \\ &= \frac{1}{R_{f,t}} Var_t^{\mathbb{Q}}(R_T) - Cov_t^{\mathbb{P}}(M_T R_T, R_T). \end{aligned} \tag{1}$$

The first component, the risk-neutral variance, can be computed directly given time-t prices of index options, starting from the work of Breeden and Litzenberger (1978). The second component is a covariance term. We use the superscript $\mathbb{P}$ to highlight the fact that those quantities are under the real-world probability measure. Henceforth, we drop the superscript and use $\mathbb{E}_t(.)$ to be the mean conditional expectation under the $\mathbb{P}$-measure.

Martin (2017) imposes a weak restriction that is termed the *negative correlation condition* (NCC). He further shows that NCC holds theoretically under mild conditions in a variety of asset pricing settings, and it also holds empirically if a typical factor structure for the SDF is assumed.

Definition 1. *The negative correlation condition (NCC) holds if*

$$Cov_t\left(M_T R_T, R_T\right) \leq 0,$$

for all M_T under the real-world probability measure.

By NCC, the risk-neutral variance can be viewed as a lower bound of the equity risk premium (the expected market excess return), which is

$$\mathbb{E}_t\left(R_T\right) - R_{f,t} \geq \frac{1}{R_{f,t}} Var_t^{\mathbb{Q}}\left(R_T\right). \tag{2}$$

Martin (2017) provides the first test on whether the implied risk-neutral volatility bounds could be related directly to the equity premium, going beyond early related studies by Merton (1980); Black (1993); Elton (1999). However, the academic study on the lower bound of the equity premium remains controversial due to the fact that the NCC is pivotal to obtaining the lower bound and yet there is no direct quantification of the premise of the NCC. Bakshi, Crosby, Gao, and Zhou (2021) exploits theoretical and empirical constructions to challenge the hypothesis of the NCC. They use options on the S&P 500 index and STOXX 50 equity index, and the tests favor rejection.

Back et al. (2022) recently test those lower bounds at different horizons conditionally and reject that they are tight for market risk premium. Therefore, using the lower bounds as forecasts of market risk premium appears insufficient in many cases due to their high slackness. Indeed, Goyal, Welch, and Zafirov (2021) reexamine those option bounds and demonstrate that the out-of-sample performance is never statistically significant. As a result, Back et al. (2022) propose to add past mean slackness to Martin's (2017) option bounds as a potential solution but is impeded by the lack of enough data to estimate mean slackness. They stress that 150 years of data are necessary for the 'bound + mean slackness' strategy to achieve a substantial improvement in out-of-sample performance.

2.2 A Generalization of Martin (2017) Bound

The significant slackness in the lower bound derived by Martin (2017) results in limited importance of the bound. Next, we generalize Martin (2017) bound for an economy where asset prices depend on a vector x_t of state variables (possibly non-fundamentals such as sentiment (e.g. Asriyan, Fuchs, and Green, 2019; Hore, 2015)). In such an economy, for any security with a return process R_t (not just the market portfolio), in equilibrium we have $E[R_T] = f(x)$, for some function $f(.)$ and that $E[M_T R_T^2] = g(x)$, for some function $g(.)$. Therefore, we have

$$E[R_T] = k(x)E(M_T R_T^2), \tag{3}$$

where $k(x) \equiv f(x)/g(x)$. It is straightforward to show that the NCC assumption in Martin (2017) (and thus Martin's lower bound) is equivalent to assuming $k(x) \geq 1$.

It follows from Equation (3) that we can obtain a new relation for the market risk premium:

$$\mathbb{E}_t(R_T) - R_{f,t} = k(x)\left(\frac{1}{R_{f,t}} Var_t^{\mathbb{Q}}(R_T) + R_{f,t}\right) - R_{f,t}, \tag{4}$$

which links the risk-neutral option bound to the state variable vector. When $k(x) \geq 1$, Equation (4) reduces to Martin's bound.

Equation (4) essentially combines the forward-looking feature (option theory) and backward-looking feature (state variable). The function form for $k(x_t)$ can be either linear or non-linear. Compared with Back et al. (2022), instead of adding the past mean values as a correction for slackness, here we use the state variables, for instance, investor sentiments, as a real-time correction to the option bound. In the next section, we empirically test the efficacy of Equation (4) in out-of-sample forecasting of market risk premium. We also compare both statistical and economic performances with those using option bounds and traditional predictors alone.

3 Econometric Methodology

In this section, we first discuss three categories of predictors used to forecast the market return, including option bounds, stock market (time-series) predictors, and the predictors that combine the first two; we next discuss the forecast construction and the criteria used to evaluate the out-of-sample forecasts.

3.1 Predictors

We consider three categories of predictors used for market risk premium forecast, namely, option bounds, stock market predictors, and the combined predictors that incorporate the stock market predictor into the risk-neutral option bounds.

3.1.1 Option bounds

We follow Martin (2017) to compute the option bounds of different horizons $T-t$,

$$b_t \equiv \frac{1}{R_{f,t}} Var_t^{\mathbb{Q}}(R_T) = (T-t) R_{f,t} \mathrm{SVIX}_{t\to T}^2, \tag{5}$$

where $\mathrm{SVIX}_{t\to T}^2$ is defined via the formula,

$$\mathrm{SVIX}_{t\to T}^2 = \frac{2}{(T-t)R_{f,t}S_t^2}\left[\int_0^{F_{t,T}} \mathrm{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \mathrm{call}_{t,T}(K)dK\right], \tag{6}$$

where $\text{put}_{t,T}(K)$ ($\text{call}_{t,T}(K)$) denotes the market price of a put (call) option with strike K and maturity $T-t$, and $F_{t,T}$ is the forward price of the underlying.

We also consider Back et al. (2022) and compute the slackness-adjusted option bounds

$$b_t + \text{mean slackness}, \tag{7}$$

where the slackness is simply the realized market excess return minus Martin's (2017) bound.

We use option price data from OptionMetrics to construct time series of option bounds at time horizons $T-t = 1, 3, 6$, and 12 months, from January 4, 1996 to December 31, 2020. We interpolate the bound linearly to match maturities of 30, 90, 180, and 360 days. To compute the slackness-adjusted bounds, we follow Back et al. (2022) to match these option bounds with realized market excess returns on S&P 500 index compounded over the 21, 63, 126, and 252 trading days. The S&P 500 daily return data are obtained from CRSP.

3.1.2 Stock market predictors

Studies investigating the time-series return predictability attempt to shed light on a variety of economic and financial variables that affect the market risk premium. However, Welch and Goyal (2008) find that a large number of predictor variables fail to outperform the historical mean in the out-of-sample tests, including the dividend-to-price ratio, book-to-market ratio, inflation, and others.

In the spirit of behavioral finance, we consider three sentiment-based variables as potential predictors as investor sentiment can generate return predictability (see, for example, De Long, Shleifer, Summers, and Waldmann, 1990). Rapach and Zhou (2022) provide a recent survey on the use of sentiment in return prediction. Specifically, we consider the sentiment index by Baker and Wurgler (2006), the sentiment index by Huang et al. (2015), and the short interest index by Rapach et al. (2016).

3.1.3 Combined predictors

Following Equations (4) and (5), we construct a *combined predictor* such that,

$$b_t\left[k(x_t)\right] = k(x_t)\left[(T-t)R_{f,t}\text{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}, \tag{8}$$

where $k(x_t) = \exp(a + bx_t)$, and x_t denotes one of the three sentiment-based predictors above.[1] We consider alternative function forms for $k(.)$ in Section 5.

One thing to notice is that option bounds with maturities of 30, 90, 180, and 360 days are computed at a daily frequency, whereas the stock market predictor variables are at a monthly frequency. We merge the stock market predictors with the option bounds computed at the end of each month and obtain a time series of combined predictors at a monthly frequency.

Taken together, we have three categories of predictors. We next test whether those predictors can successfully predict the market excess returns out of sample.

3.2 Forecast construction

We employ the out-of-sample tests since such tests provide the most rigorous and relevant evidence on stock return predictability (Welch and Goyal, 2008; Martin and Nagel, 2022; Dong et al., 2022). Because of the horizon-matching feature in Martin's (2017) theory, we cannot focus on the monthly, quarterly, or annual regressions as in the conventional literature. Instead, our main results are based on market risk premium forecast over the next 21 trading days horizon, thus with the 30-day option bound. Same procedure is implemented by Back et al. (2022). For brevity, we name it the 1-month forecast. We consider longer horizons in Section 5.

For time-series predictors and combined predictors, we begin with a standard predictive regression model,

$$r_{t+1} = \alpha_t + \beta_t Z_{i,t} + \varepsilon_{t+1}, \tag{9}$$

[1]For the two parameters $\{a, b\}$, we generate the uniformly-distributed random variables in each forecast.

where r_{t+1} is the market excess return over 21 trading days, $Z_{i,t}$ is a predictor, and ε_{t+1} is a disturbance term. As in Welch and Goyal (2008); Campbell and Thompson (2008); Rapach et al. (2010), we generate out-of-sample forecasts using a recursive estimation window such that,

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t Z_{i,t}, \tag{10}$$

where $\{\hat{\alpha}_t, \hat{\beta}_t\}$ are ordinary least squares (OLS) estimates using the data up to time t.

For option bounds, we have to deviate from the standard procedure. Under the option theory by Martin (2017), the option bound is already a meaningful expected return. Therefore, we directly use the option bound as a forecast. The same argument can be applied to the slackness-adjusted option bound by Back et al. (2022). Thus,

$$\hat{r}_{t+1} = b_t, \quad \text{or} \quad \hat{r}_{t+1} = b_t + \text{mean slackness}. \tag{11}$$

Despite the theory, we can still treat the option bound, b_t, as a predictor and run a predictive regression to predict the future return,

$$\hat{r}_{t+1} = \hat{\alpha}_t + \hat{\beta}_t b_t. \tag{12}$$

We assess the forecast accuracy with an out-of-sample R^2 statistic, namely, R^2_{OS}, relative to the prevailing historical average.[2] Given T forecasts in the out-of-sample evaluation period, R^2_{OS} statistic essentially measures the relative reduction in mean square prediction error (MSPE),

$$R^2_{OS} = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \bar{r}_t^{HA})^2}. \tag{13}$$

where $\bar{r}_t^{HA}$ and $\hat{r}_t$ denote the predicated market excess returns based on the historical mean and a competing model, respectively. When $R^2_{OS} > 0$, the $\hat{r}_t$ forecast generates a lower MSPE than the

[2]The historical average of market risk premium is constructed from the daily return series of S&P 500 index starting from July 3, 1962 on CRSP.

prevailing mean forecast, delivering out-of-sample evidence of return predictability.

To assess the statistical significance of R^2_{OS}, we use the Clark and West's (2007) *MSPE-adjusted* statistic. Traditional predictors typically perform poorly in out-of-sample forecasts with negative R^2_{OS}, suggesting that the historical average forecast is difficult to beat (Welch and Goyal (2008)). Additionally, because market excess returns consist of a large unpredictable component, the R^2_{OS} is usually small. Notwithstanding, Campbell and Thompson (2008) suggest that a monthly R^2_{OS} statistic of 0.5% is the threshold for economic significance for a mean-variance investor. In Section 4.5, we directly assess the economic values of various market risk premium predictors by measuring their economic values to an investor.

Our sample spans from January 1996 to December 2020 due to the option data availability. We consider three different out-of-sample forecast evaluation periods: (i) 2001:01–2020:12 with an initial estimation window of 5 years; (ii) 2006:01–2020:12 with an initial estimation window of 10 years; and (iii) 2011:01–2020:12 with an initial estimation window of 15 years. Overall, considering multiple out-of-sample periods helps provide us with a good sense of the robustness of the out-of-sample forecasting results.

4 Out-of-Sample Results

In this section, we present our main out-of-sample results, the efficacy of imposing economic restrictions, possible statistical explanations, and finally the economic values of combing option prices with stock market predictors.

4.1 Statistical gains

Table 1 reports the monthly R^2_{OS} statistics (in percentage) for forecasting future 1-month market excess returns for the three evaluation periods. Panels A, B, and C present the results of option bounds, sentiment-based predictors, and the combined predictors that incorporates option prices into each of the three sentiment-based measures, respectively.

In Panel A, we find that option bounds perform poorly in out-of-sample tests. Using Martin's (2017) option bound directly as a forecast of future market risk premium produces either a negative or positive but insignificant R^2_{OS} statistics. For example, we find that option bound delivers a monthly R^2_{OS} of -1.450% during the evaluation periods between 2001 and 2020. Moreover, using option bounds as regression predictors lead to much worse out-of-sample results. For instance, we find that R^2_{OS} becomes -8.445% once we run the forecast in an OLS regression. Adding the past mean slackness as a bound correction does not improve the out-of-sample performance. For instance, for the out-of-sample period of 2001–2020, the R^2_{OS} statistic is -2.533%. As argued by Back et al. (2022), the improvement from adding past mean slackness is limited due to the lack of available slackness data.[3] For instance, for the evaluation periods between 2011 and 2020, the R^2_{OS} statistics jump from 0.741% for Martin's (2017) option bound to 2.227% for Back et al. (2022) adjusted bound, though neither is statistically significant.

Panel B, Table 1 reports the results of time-series predictors. In most cases, these predictors perform poorly with negative R^2_{OS} statistics. For example, two investor sentiment indices, IS_{BW} and IS_{HJTZ}, and short interest index, *SSI* all have negative R^2_{OS} statistics of -1.443%, -0.268%, and -0.776%, respectively in the out-of-sample period 2001–2020. We only observe a significantly positive R^2_{OS} statistic for *SSI* in the evaluation periods 2006–2020, and a marginally significantly positive R^2_{OS} for IS_{BW} in the period 2011–2020. In other words, the out-of-sample predictability of traditional predictors is not robust. The lack of consistent out-of-sample evidence indicates the need for refinement of those stock market predictors.

Panel C, Table 1 reports the forecasting results of the combined predictors that incorporate both option market information and stock market information. Compared with Martin's (2017) options bounds in Panel A, the combine predictors suggest much stronger out-of-sample evidence that market return is predictable. For instance, for the period 2001–2020, combing option prices with IS_{HJTZ} and *SII* yield out-of-sample R^2_{OS} statistics of 0.809% and 0.695%, both significant at the 5%

[3] Back et al. (2022) argue that to achieve a significantly positive R^2_{OS}, researchers need at least 150 years of data to estimate the past mean slackness.

level. In the last row of Panel C, Table 1, we aggregate the individual forecasts from three combined predictors by taking an arithmetic mean. Rapach et al. (2010) show that a simple combination forecast exerts a strong shrinkage effect, and generally achieve a better forecasting performance. Indeed, We find that pooling generates substantial forecasting gains with significantly positive R^2_{OS} statistics in all evaluation periods, ranging from 0.649% to 1.649%. As pointed out by Campbell and Thompson (2008), the R^2_{OS} of 0.5% for monthly data can signal an economically meaningful degree of return predictability in terms of increased annual portfolio returns for a mean-variance investor. Therefore, combining stock market information with option market information produces economically significant gains consistently over time.

To get a better sense of those forecasts, Figure 1 depicts the out-of-sample forecasts for the 2001–2020 out-of-sample period. The blue line in each panel delineates the historical mean benchmark forecast. Both option bound forecast (Panel A) and the short interest index forecast (Panel B) are substantially more volatile and imply implausibly negative or unrealistically large values for numerous months over the out-of-sample testing period. On the contrary, the combined predictor (Panel C) generates relatively less volatile forecast, and pooling (Panel D) further shrinks the forecast. It seems that combining two markets' information better regulates the forecast variability. We conduct more statistical tests shortly.

Overall, we show that combining information from both derivative market and stock market significantly improve market risk premium forecasts versus using either type of information alone.

4.2 Upper bounds

In this subsection, following Campbell and Thompson (2008); Pettenuzzo et al. (2014), we consider imposing a simple economic upper bound on our forecasts. The Sharpe ratio reported in the previous section corresponds to the *ex-post* measure of the realized portfolio returns in the out-of-sample period. We next consider the *ex-ante* Sharpe ratio perceived by the investor at time t

$$SR_{j,t} = \frac{\hat{r}_{j,t+1}}{\hat{\sigma}_{t+1}}, \tag{14}$$

where $\hat{r}_{j,t+1}$ is the forecasted excess return on S&P 500 index based on strategy j at time t, and $\hat{\sigma}_{t+1}$ is the forecasted volatility computed from historical return using a 5-year rolling window.

Figure 2 plots the distribution of the *ex-ante* Sharpe ratios from Martin's (2017) bound, combining bound with the aggregate short interest index (*SII*), and the pooling. On average, the *ex-ante* annualized Sharpe ratio is around 0.2 for Martin (2017), 0.75 for pooling, and 1 for the combined predictor of bound and *SII*, respectively. The large Sharpe ratios from our combined predictors and pooling forecasts suggest a potential improvement by imposing an upper bound on the Sharpe ratio. Hence, based on MacKinlay (1995) and Cochrane and Saa-Requejo (2000), we use a value varying from 0.6 to 1 to bound the above Sharpe ratio, yielding a new and economic constrained forecast. Next, we repeat our out-of-sample one-month market risk premium forecasts for the combined predictors in Table 1 using 0.6 and 1 as the maximum Sharpe ratio, respectively. Specifically, we truncate the forecast to have an annualized Sharpe ratio of 0.6 (1) if the ratio of forecasted return to forecasted volatility (annualized) is greater than 0.6 (1).

Panel A of Table 2 reports the out-of-sample R^2_{OS} for our combined predictors with an upper bound on the maximum Sharpe ratio of 0.6 for various evaluation periods. Compared with results without restrictions on upper bounds, we deliver considerably stronger results with larger R^2_{OS} statistics in all evaluation periods. For example, for the evaluation periods between 2001–2020, the R^2_{OS} statistic increases from 0.809% (0.695%) in Table 1 to 2.383% (2.148%) when combing IS_{HJTZ} (*SII*) with option bounds. In addition, Pooling delivers an out-of-sample R^2 statistic as high as 2.199%, as opposed to 1.417% in Table 1 when no economic restriction is imposed. We find similar results in other out-of-sample testing periods.

Panel B of Table 2 reports the results of the out-of-sample forecasts with an upper bound of 1 for the Sharpe ratio. We also observe an improvement in out-of-sample forecasts in each evaluation period. For example, the new forecast of combing *SII* with option bounds yields an R^2_{OS} of 1.718% relative to 0.695% in Table 1 for the evaluation periods between 2001–2020. Collectively, we show a significant improvement in the out-of-sample performance by imposing an economically reasonable upper bound on the maximum Sharpe ratio.

4.3 Forecast stabilization

In this section, we conduct bias-variance analysis in order to provide some statistical explanations for the superior performance of combing option bounds with stock market predictors. Since the R^2_{OS} statistic essentially compares the MSPEs between two forecasting approaches in Equation (13), we follow Theil (1966) to decompose MSPE as follows,

$$MSPE = \left(\bar{\hat{e}}\right)^2 + Var(\hat{e}), \tag{15}$$

where $\hat{e}$ signifies the forecast error, $\left(\bar{\hat{e}}\right)^2$ is the squared forecast bias, and $Var(\hat{e})$ is the forecast variance. Thus, a decrease in forecast variance based on combining forecasts can help to reduce MSPE and thereby increase the out-of-sample R^2_{OS} statistics as long as combining forecasts do not come with a large increase in bias.

Figure 3 provides three scatterplots depicting the forecast variance and the squared forecast bias for the out-of-sample forecasts based on historical mean, option bounds, stock market predictors, and combined predictors for the period 2001–2020. In Panel A, the forecasts of option bounds all display much higher forecast variance than the historical average benchmark. In Panel B, compared with the historical mean forecast, investor sentiment both display higher forecast variance and higher squared forecast bias. Although using short interest index slightly reduces the forecast variance, the corresponding squared forecast bias increases by 8 times relative to the benchmark. As a result, neither option bounds nor stock market predictors generates smaller MSPEs relative to the benchmark, and thus negative R^2_{OS} values in Table 1.

Panel C presents the bias-variance decomposition for the three combined predictors and the pooling of the three forecasts. We find that combining option bounds with either IS_{HJTZ} or IS_{SII} exhibits much lower forecast variance relative to the benchmark. Despite a larger squared forecast bias, the reduction in forecast variance outweighs the increase in squared forecast bias. As a result, they both produce good out-of-sample performance relative to the historical mean forecast. Interestingly, we also find that pooling the individual forecasts reduces both squared forecast bias and forecast variance, which leads to a much smaller MSPE and thereby a much larger R^2_{OS} statistics. This finding is consistent with Rapach et al. (2010) that pooling can effectively regularize the forecast variability, thereby generating substantial forecasting gains consistently over time. In summary, combining information on option bounds and investor sentiment reduces forecast variance, thereby improving out-of-sample forecasts.

4.4 Encompassing test

We next compare the information content in different forecasts using the forecast encompassing tests developed by Chong and Hendry (1986); Fair and Shiller (1990). Consider an optimal composite forecast of r_{t+1} as a convex combination of forecasts from two models, i and j,

$$\hat{r}^{\star}_{t+1} = (1-\lambda)\hat{r}_{i,t+1} + \lambda\hat{r}_{j,t+1}, \quad 0 \leq \lambda \leq 1. \tag{16}$$

If $\lambda = 0$, this suggests that model j does not carry any useful information and thus its forecast is encompassed by model i. Conversely, if $\lambda > 0$, the forecast of model i does not encompass the model j forecast because of useful information contained in model j. Thus, forecast encompassing tests indicate that it is useful to combine forecasts from models i and j compared with using solely model i if we reject the null hypothesis of encompassing.

To test the null hypothesis that model i encompasses j ($H_0 : \lambda = 0$), against the one-sided alternative hypothesis that the model i does not encompass j ($H_1 : \lambda > 0$), we follow Harvey, Leybourne, and Newbold (1998) to compute the modified HLN-statistic over the out-of-sample evaluation period of T_0. Define $d_{t+1} = (\hat{e}_{i,t+1} - \hat{e}_{j,t+1})\hat{e}_{i,t+1}$, where $\hat{e}_{i,t+1} = r_{t+1} - \hat{r}_{i,t+1}$ and $\hat{e}_{j,t+1} = r_{t+1} - \hat{r}_{j,t+1}$. Let $\bar{d} = \frac{1}{T_0}\sum_k d_k$, and we compute

$$MHLN = \frac{T_0 - 1}{T_0}\left(\left[\hat{V}(\bar{d})\right]^{-\frac{1}{2}}\bar{d}\right) \sim t_{T_0-1}, \tag{17}$$

where $\hat{V}(\bar{d}) = \frac{1}{T_0}\hat{\phi}_0$ and $\hat{\phi}_0 = \frac{1}{T_0}\sum_k (d_k - \bar{d})^2$.

Table 3 reports the Harvey et al.'s (1998) *MHLN* statistic p-values applied to the evaluation periods between 2001 and 2020. Each entry in the table corresponds to the null hypothesis that the forecast estimated in the row heading is encompassed by the forecast based on the column heading. We reject the null hypothesis that the option bound forecasts encompass the combined predictors' forecasts. For example, in the third column, the p-value of "Bound & IS_{HJTZ}" is 0.03 and statistically significant. We also observe highly significant results for "bound + slackness" and "Bound (OLS)" forecasts. Additionally, we can reject the null hypothesis that the traditional predictors' forecasts encompass the combined predictors' forecasts at the 5% level. By contrast, we can not reject the null hypothesis that combined predictors' forecasts encompass either option bounds' forecasts or traditional predictors' forecasts.

In summary, the above forecast encompassing tests justify using information from both derivative market and stock market in equity risk premium forecast.

4.5 Economic values

Apart from the statistical accuracy, we next compare the benchmark and competing forecasts in terms of their economic values to an investor. Specifically, consider a mean-variance investor who allocates across equities and a risk-free asset (the Treasury bill) each month. At the end of month t, the investor faces the objective function

$$\arg_{\hat{\omega}_{t+1}} \hat{\omega}_{t+1}\hat{r}_{t+1} - \frac{\gamma}{2}\hat{\omega}^2_{t+1}\hat{\sigma}^2_{t+1}, \tag{18}$$

where γ denotes the coefficient of relative risk aversion, $\{\hat{\omega}_{t+1}, 1-\hat{\omega}_{t+1}\}$ are allocation weights to the market portfolio and the risk-free asset at month $t+1$, $\hat{r}_{t+1}$ is the investor's market excess return forecast, and $\hat{\sigma}^2_{t+1}$ is the investor's forecast of the variance of the market excess return. The optimal mean-variance portfolio weight on the market can be computed as

$$\hat{\omega}^{\star}_{t+1} = \left(\frac{1}{\gamma}\right)\left(\frac{\hat{r}_{t+1}}{\hat{\sigma}^2_{t+1}}\right). \tag{19}$$

We follow Campbell and Thompson (2008) to set γ to be 3 and to constrain the portfolio weight on stocks to lie between $[0, 1.5]$ each month in Equation (19). Over the out-of-sample periods, we compute four quantities (performance measures), based on the mean $\hat{\mu}_j$ and standard deviation $\hat{\sigma}_j$ of the out-of-sample realized returns by a forecasting method j. First, we measure the *out-of-sample Sharpe ratio* (SRatio)

$$\hat{s}_j = \frac{\hat{\mu}_j}{\hat{\sigma}_j}. \tag{20}$$

To test whether the Sharpe ratios of the two strategies are statistically distinguishable, we follow DeMiguel, Garlappi, and Uppal (2009) to compute the p-value of their difference.

Second, we compute the *certainty-equivalent return* (CER) of each strategy,

$$C\hat{E}R_j = \hat{\mu}_j - \frac{\gamma}{2}\hat{\sigma}_j^2. \tag{21}$$

Relative to a benchmark, we also compute the CER difference, which is known as the utility gain in the forecasting literature (see, e.g., Rapach and Zhou, 2022).

Next, we compute DeMiguel et al.'s (2009) *return-loss value* with respect to Rapach et al.'s (2010) simple pooling. Precisely, suppose $\{\hat{\mu}_b, \hat{\sigma}_b\}$ are the monthly out-of-sample mean and volatility of the excess returns from the benchmark, the return-loss from the competing forecast j is

$$\text{return-loss}_j = \left(\frac{\hat{\mu}_b}{\hat{\sigma}_b}\right) \times \hat{\sigma}_j - \hat{\mu}_j. \tag{22}$$

In other words, the return-loss is the additional return needed for strategy j to perform as well as the benchmark. Therefore, a negative return-loss value indicates that the method j outperforms the simple pooling in terms of the Sharpe ratio.

Lastly, we compute the *performance fee* suggested in Fleming et al. (2001). It can be interpreted as the maximum fee that a quadratic-utility investor would be willing to pay to switch from the benchmark to the alternative. To estimate this fee, we find the value of Δ that solves

$$\sum_t \left[\left(R_{j,t} - \Delta\right) - \frac{\gamma}{2(1+\gamma)} \left(R_{j,t} - \Delta\right)^2 \right] = \sum_t \left[R_{b,t} - \frac{\gamma}{2(1+\gamma)} R_{b,t}^2 \right], \tag{23}$$

where $R_{j,t}$ and $R_{b,t}$ denote the out-of-sample realized returns by the competing forecast j and the benchmark, respectively. We report the estimate of Δ as annualized fees in basis points.

Figure 4 plots log cumulative excess returns for portfolios using market excess return forecasts based on option bound, short interest index, and the combined predictor. The figure also depicts the cumulative excess return for the portfolio based on the historical average benchmark. Figure 4 reveals that the portfolio that only rely on option bounds underperforms the portfolio based on the historical mean benchmark, with dramatic loss during financial crisis in 2008/09. In contrast, the portfolio that only rely on short interest index outperforms the benchmark. However, the portfolio that incorporate the information from both options and short interest index exhibits even better performance. Moreover, pooling across individual forecasts from the combined predictors better gauges against the downside risk during the market stress, and at the same time, maintain the upside potential.

Table 4 reports the above performance measures on returns over the period 2001–2020. All results are annualized. In Panel A, we report the benchmark value when using historical average forecast, including average excess return, standard deviation, Sharpe ratio, and certainty equivalent return. We find that in Panel B, the forecasts based on Martin (2017) option bound, and Back et al. (2022) slackness-adjusted bound produce standard deviations twice the magnitude of the benchmark, leading to a much smaller Sharpe ratio and negative CEQ. The positive return-loss and negative performance fees both suggest that investors strongly prefer the historical average benchmark to the option bound when forecasting the market risk premium. This finding is consistent with the negative out-of-sample R^2_{OS} statistics documented in Table 1.

Panel C and D in Table 4 report the performance measures based on forecasts from stock market predictors, and from the combined predictors. We find that using stock market predictors alone produce reasonably large economic values relative to the benchmark, in terms of larger Sharpe ratios, larger CEQs, negative return-loss values, and positive performance fees. Moreover, combining information from both stock market and option market generate even larger forecasting gains versus using either information alone. For example, relative to the Sharpe ratio of 0.212

from the historical mean benchmark, short interest index produces a Sharpe ratio of 0.435, twice the magnitude and the difference is marginally significant at the 10% level. Moreover, combing short interest index with option bounds yields a Sharpe ratio as high as 0.525, an additional 20% increase relative to short interest index alone. The Sharpe ratio difference relative to the benchmark also becomes more significant at the 5% level. Furthermore, the performance fees jump from 189 bps to 385 bps once we change from using short interest alone to using the combined predictor. The high performance fee suggest that the investor is willing to pay around 385 bps per annum to acquire the information from two markets. The results are robust to a proportional transaction cost.

Overall, Table 4 highlights the investment value by combing the forward-looking (option data) and backward-looking (sentiment variables) features in forecasting the market risk premium.

5 Robustness

In this section, we provide robustness checks for our out-of-sample tests. We separately run our out-of-sample forecasts based on NBER-dated business-cycles. We conduct long-horizon return forecasts. We also present our results using alternative function forms or extended data sample.

5.1 Violation of NCC

We next present time-series plots of the cumulative differences in squared forecast errors for the historical average benchmark relative to each competing forecast for the period 2001–2020 (Welch and Goyal, 2008; Rapach et al., 2010).

$$\text{square error difference} = (r_t - \bar{r}^{HA}_{t|t-1})^2 - (r_t - \hat{r}_{t|t-1})^2, \tag{24}$$

where $\bar{r}^{HA}_{t|t-1}$ is the historical average (HA), r_t is the realized market excess return, and $\hat{r}_{|t-1}$ is the forecast based on a competing model.

The cumulative differences in Figure 5 make it straightforward to determine whether a competing forecast is more accurate than the benchmark for any subsample. We simply compare the height of the curve at the start and end of the subsample. If the curve is higher (lower) at the end, then the competing forecast has a lower (higher) MSFE than the benchmark over the subsample. A predominantly positively sloped curve essentially suggests a competing forecast that provides out-of-sample gains on a consistent basis, whereas a steeply negatively sloped segment indicates an episode of severe underperformance. Forecasts by either option bound or short interest index in Figure 5 fails to provide accuracy gains on a reasonably consistent basis over time. Combing option bound with short interest index improves the accuracy gains as the curve becomes positive since 2010. Further, pooling across individual forecasts from combined predictors generate much higher accuracy gains, as the cumulative curve stay above zero after 2009, and ends up at a higher point at the end of the testing period.

Notably, the failure of option bound by Martin (2017) is mainly due to the recessions around 2008/09 global financial crisis. The option bound substantively underperform the historical average benchmark. One plausible explanation is that the efficacy of Martin (2017) bound depends on the NCC. However, as plotted in Figure 6, we find that the NCC is violated frequently during the evaluation period between 2001–2020, with most cases concentrated either in high investment sentiment periods or during market recessions, for instance, the 2008/09 crisis. The demand for options (as a hedging tool) surges during severe market periods, and as a result, the option prices increase substantially. Recall that the option bound is essentially a weighted average of market prices of index options. Therefore, simply using the option bound as a "meaningful expected return" as in Martin (2017) may leads to unrealistic positive forecast during the market stress. In other words, the pivotal assumption behind Martin (2017) approach is violated.

5.2 NBER-dated business-cycle

In this subsection, we analyze the forecast during the NBER-dated expansions and recessions. Specifically, we compute the R^2_{OS} statistics separately for the NBER-dated expansions and recessions within the out-of-sample testing periods. In total, there are three recessions over the out-of-sample periods between January 2001 and December 2020, with business-cycle peaks (troughs) occurring at 2001:03 (2001:11), 2007:12 (2009:06), and 2020:02 (2020:04). Due to the limited sample size in recessions (with about 30 observations), we only assess the statistical significance of for positive R^2_{OS} statistics in expansions.

In Panel A, Table 5, we find that the R^2_{OS} statistics using option bound are more negative in recessions than in expansions, leading to negative R^2_{OS} statistics for the full sample periods. Panel B shows that the sentiment-based predictors perform poorly with large negative R^2_{OS} statistics during recessions, while they perform generally positive and significant R^2_{OS} during expansions. As a result, the R^2_{OS} statistics for the full sample periods all become negative. For example, IS_{HJTZ} produces a marginally significant R^2_{OS} statistic of 0.307% during expansions, but a negative R^2_{OS} of -1.332% during recessions. In Panel C, we find that the combined predictors and pooling all yield positive R^2_{OS} statistics during both expansions and recessions (with one exception for *SSI*). For example, when combining option bound with IS_{HJTZ}, the R^2_{OS} statistics are 0.814% and 0.800 for expansions and recessions, ending up with a significantly positive R^2_{OS} of 0.809% for the full out-of-sample period.

In summary, combing option bounds with stock market predictors is more likely to recover the market risk premium predictability in both expansion and recession periods, thereby outperforming the historical average benchmark overall.

5.3 Long-horizon forecasts

In this subsection, we repeat the out-of-sample tests using various long horizons. As discussed before, we cannot focus on monthly, quarterly, or annual regressions as in the literature because of the horizon-matching feature in Martin's (2017) theory. Since the option bounds can be computed for alternative horizons by matching option maturities, we examine the out-of-sample forecasting of 3-month, 6-month, and 12-month horizons.

Table 6 reports the out-of-sample R^2_{OS} statistics for the evaluation period 2001–2020. For the 3-month horizon forecast, option bounds fail to outperform the historical average forecast with R^2_{OS} statistics either being negative or positive but insignificant. In contrast, stock market predictors begin to outperform the historical mean benchmark forecast with significantly positive R^2_{OS} values, ranging from 1.181% to 3.471%. Additionally, combined predictors of IS_HJTZ, SSI and pooling generate even better out-of-sample performance relative to the benchmark forecast, with significantly positive R^2_{OS} statistics ranging from 2.435% to 6.065% for the 3-month horizon forecasts.

In general, out-of-sample forecasting performs better for longer horizons. For semi-annual and annual forecasting, both option bounds and traditional predictors yield positive and significant R^2_{OS} statistics. This finding is anticipated as the market, in general, displays an upward trend in the long run. Similar results are obtained by Campbell and Thompson (2008) when they forecast annual market return using macro variables (Panel B, Table 2, in their paper). Despite the improved performance, combing the two information sets still produces better results, especially when we apply the pooling method. For instance, pooling generates R^2_{OS} statistics as high as 14.105% and 26.975%, for the semi-annual and annual horizons, respectively. Overall, we show that our combined predictors consistently perform better than using either information alone in different forecast horizons.

5.4 Alternative function forms

So far, we have demonstrated that the generalization of Martin (2017) bound by incorporating stock market variables generates superior out-of-sample forecasting gains. In Equation (8), we rely on an exponential form for the function $k(.)$. In this subsection, we consider two alternative function forms $k(x_t)$. We then repeat the out-of-sample forecasts at 1-, 3-, 6- and 12-month horizons using the new combined predictors.

Case 1: Exponential form with individual terms. Equation (8) only scales and operates through interaction. We modify $k(x_t)$ to incorporate both interaction and individual terms[4]

$$b_t\left[k(x_t)\right] = \Big[\underbrace{\exp(ax_t + b) + 1}_{k(x_t)}\Big] \times \left[(T-t)R_{f,t}\mathrm{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}. \tag{25}$$

Case 2: Linear form. We consider a linear function form for $k(x_t) = ax_t + b$ so that,

$$b_t\left[k(x_t)\right] = \Big[\underbrace{a + bx_t}_{k(x_t)}\Big] \times \left[(T-t)R_{f,t}\mathrm{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}. \tag{26}$$

The results are presented in Table 7. We find consistent results with these in Tables 1 and 6. These alternative combined predictors significantly outperform the historical mean forecast at various horizons. The out-of-sample R^2 statistics are statistically significant at the 5% level or better in most cases. For example, after incorporating the short interest index into option bounds, we obtain R^2_{OS} statistics of 0.448%, 3.575%, 8.764%, and 14.387% at 1-, 3-, 6-, and 12-month horizons, respectively. Moreover, the pooling forecasts consistently outperform the prevailing mean at various horizons with R^2_{OS} statistics, ranging from 1.325% to 26.456%. Overall, the generalization of Martin (2017) bound is robust to alternative function forms to combine information from option prices and state variables.

5.5 Longer sample period

So far, our sample starts from 1996 due to the option data availability. The option bounds and our combined predictors both rely on the construction of SVIX from market prices of index options. Since SVIX and the publicly traded VIX index are known to be highly correlated (with a correlation coefficient as high as 0.99), we collect the VIX data from CBOE starting from 1990,

[4]We thank Tyler Beason for this great suggestion.

and reconstruct the combined predictor such that,

$$b_t\left[k(x_t)\right] = \begin{cases} k(x_t)\left[(T-t)R_{f,t}\text{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}, & \text{if} \quad t \geq 1996, \\ k(x_t)\left[(T-t)R_{f,t}\text{VIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}, & \text{if} \quad t < 1996 \end{cases} \tag{27}$$

Recall that,

$$\text{SVIX}^2_{t\to T} = \frac{2}{(T-t)R_{f,t}S_t^2}\left[\int_0^{F_{t,T}} \text{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \text{call}_{t,T}(K)dK\right]. \tag{28}$$

Similarly, we can write down the formula for VIX based on index option prices,

$$\text{VIX}^2_{t\to T} = \frac{2R_{f,t}}{(T-t)}\left[\int_0^{F_{t,T}} \frac{1}{K^2}\text{put}_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} \frac{1}{K^2}\text{call}_{t,T}(K)dK\right]. \tag{29}$$

VIX and SVIX both capture important aspects of market return, and the difference between two indices can be quite small under certain circumstances, for instance, log-normality (see Martin, 2017). Figure 7 depicts the 30-day SVIX index and the CBOE VIX index. Two indices exhibit almost the same pattern. Compared to VIX, SVIX measures the risk-neutral volatility, whereas VIX measures the risk-neutral entropy such that,

$$VIX^2_{t\to t+T} = \frac{2}{T}L_t^Q\left(\frac{R_{t\to t+T}}{R_{f,t\to t+T}}\right), \tag{30}$$

where $L_t^Q(X) \equiv log\mathbb{E}_t^Q X - \mathbb{E}_t^Q logX$.

We extend the full sample to cover from January 1990 to December 2020. Apart from the three out-of-sample periods used before, we choose one more out-of-sample period: 1996:01–2020:12. The R^2_{OS} statistics are presented in Table 8. Consistent with our main results, combing option theory with traditional predictors deliver a much accurate forecasting on a consistent basis than using either option bounds or stock market predictors alone.

6 Conclusion

Predicting market risk premium, or expected market excess return, is one of the fundamental problems in finance because the market risk premium is a key determinant of the required rate of returns for investors to hold assets in asset pricing models. The out-of-sample predictability, however, remains small, as shown by studies in the time-series predictability literature. Recent developments by Martin (2017) and others shed light on the expected market excess return from options prices, but the empirical performance is still unsatisfactory.

In this paper, we provide the first study on combining two lines of literature on market risk premium. We theoretically derive a new bound on the market risk premium by combining risk-neutral variance with state variables. We further show that the new combined predictor performs well empirically and the improvement in out-of-sample forecasting is economically substantial. Collectively, our paper provides new insights on the market risk premium by drawing our understandings from both the time-series return predictability literature and the option literature.

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Table 1: R^2_{OS} statistics (in percent) for 1-month market risk premium forecast

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 1-month market excess return forecasts based on option bounds, time-series (stock market) predictors, and combined predictors. The combined predictor takes the form such that,

$$b_t\left[k(x_t)\right] = k(x_t)\left[(T-t)R_{f,t}\text{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t},$$

where $R_{f,t}$ is the gross return on an risk-free asset, $\text{SVIX}^2_{t\to T}$ is computed from market prices of index options as in Martin (2017), x_t is one of time-series variables, and $k(x_t) = exp(a + bx_t)$. In the last row, we pool the individual forecasts from combined predictors by taking an arithmetic mean.

Panel A: Option bounds is when $k(x_t) = 1$. We use the bound as a direct measure of market excess return. Alternatively, we also use the bound as a predictor in predictive regression, which we label as "Bound (OLS)". Besides, we also use the past mean slackness, which is the realized market excess return minus the bound, as a correction to the bound, as suggested by Back et al. (2022).

Panel B: Time-series predictor is when $b_t\left[k(x_t)\right] = x_t$. We use Baker and Wurgler's (2006) sentiment index (IS_{BW}), Huang et al.'s (2015) sentiment index (IS_{HJTZ}), and Rapach et al.'s (2016) short interest index (SSI).

The out-of-sample periods are 2001:01–2020:12, 2006:01–2020:12, and 2011:01–2020:12, as indicated in column headings. Based on the Clark and West's (2007) test, * and ** indicate significance at the 10% and 5% levels for the positive R^2_{OS}, respectively.

Out-of-sample periods	2001−2020	2006−2020	2011−2020
Panel A: Option bounds			
Bound	−1.450	−2.214	0.741
Bound + Slackness	−2.533	−2.439	2.227
Bound (OLS)	−8.445	−9.519	−3.206
Panel B: Time-series predictors			
$x_t = IS_{BW}$	−1.443	−0.401	1.869*
$x_t = IS_{HJTZ}$	−0.268	0.085	0.860
$x_t = SSI$	−0.776	1.520**	−0.553
Panel C: Combined predictors			
$x_t = IS_{BW}$	−0.788	−0.481	1.717*
$x_t = IS_{HJTZ}$	0.809**	−0.632	1.504*
$x_t = SSI$	0.695**	1.822**	0.186*
Pooling	1.417**	0.649*	1.649*

Table 2: R^2_{OS} statistics (in percent) with upper bounds

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 1-month market excess return based on the combined predictor such that

$$b_t\left[k(x_t)\right] = k(x_t)\left[(T-t)R_{f,t}\mathrm{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t},$$

where $\mathrm{SVIX}^2_{t\to T}$ is computed from market prices of index options as in Martin (2017), x_t is one of three sentiment variables, and $k(x_t) = exp(a + bx_t)$. We consider two Sharpe ratio values as upper bounds, as indicated in panel headings. In the last row of each panel, we pool the individual forecasts by taking an arithmetic mean. Based on the Clark and West's (2007) test, *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively.

Out-of-sample periods	2001–2020	2006–2020	2011–2020
	Panel A: Maximum Sharpe ratio of 0.6		
$x_t = IS_{BW}$	0.231	0.36	1.905**
$x_t = IS_{HJTZ}$	2.383**	0.532	1.776*
$x_t = SSI$	2.148**	2.413**	1.497*
Pooling	2.199**	1.198**	1.795*
	Panel B: Maximum Sharpe ratio of 1.0		
$x_t = IS_{BW}$	−0.627	−0.325	1.705*
$x_t = IS_{HJTZ}$	1.282**	−0.121	1.576*
$x_t = SSI$	1.718**	2.347**	1.009*
Pooling	1.774**	0.955*	1.802*

Table 3: Forecast encompassing test results, *MHLN* statistic *p*-values

This table reports p-values for the Harvey et al.'s (1998) *MHLN* statistic for the out-of-sample forecasting based on option bounds, traditional predictors, and combined predictors. The statistic corresponds to an upper-tail test of the null hypothesis that the forecast given in the column heading encompasses the forecast given in the row heading against the alternative hypothesis that the forecast given in the column heading does not encompass the forecast given in the row heading. The out-of-sample period is 2001:01–2020:12.

(1)	(2) HA	(3) Bound	(4) Bound + Slackness	(5) Bound (OLS)	(6) IS_{BW}	(7) IS_{HJTZ}	(8) *SSI*	(9) Bound + IS_{BW}	(10) Bound + IS_{HJTZ}	(11) Bound + *SSI*	(12) Pooling
HA		0.154	0.059	0.020	0.031	0.055	0.021	0.075	0.177	0.098	0.323
Bound	0.743		0.038	0.024	0.104	0.178	0.070	0.150	0.310	0.218	0.490
Bound + Slackness	0.797	0.686		0.036	0.325	0.463	0.324	0.288	0.588	0.568	0.794
Bound (OLS)	0.791	0.651	0.608		0.536	0.614	0.568	0.479	0.685	0.667	0.760
IS_{BW}	0.292	0.092	0.077	0.011		0.692	0.120	0.502	0.823	0.270	0.960
IS_{HJTZ}	0.088	0.038	0.043	0.009	0.090		0.045	0.182	0.759	0.125	0.746
SSI	0.077	0.024	0.029	0.018	0.051	0.078		0.071	0.174	0.888	0.496
Bound + IS_{BW}	0.161	0.069	0.062	0.009	0.176	0.352	0.067		0.644	0.142	0.804
Bound + IS_{HJTZ}	0.050	0.030	0.034	0.007	0.046	0.124	0.030	0.072		0.076	0.517
Bound + *SSI*	0.029	0.016	0.023	0.015	0.021	0.028	0.067	0.035	0.073		0.256
Pooling	0.028	0.017	0.021	0.008	0.007	0.033	0.030	0.030	0.127	0.100	

Table 4: Economic values

This table reports various economic measures for a mean-variance investor with a relative risk aversion coefficient of three who allocates *monthly* between equities and risk-free bills for the out-of-sample period is 2001:01–2020:12. The allocation weights depend on the return forecasts as indicated by panel headings. The performance measures include out-of-sample average excess return, standard deviation, Sharpe ratio (SRatio), certainty equivalent return (CER), DeMiguel et al.'s (2009) return-loss value, and Fleming et al.'s (2001) performance fee (Fee). For SRatio and CER, we also compute the difference between any competing models and the historical mean benchmark. T All results are annualized. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

(1)	(2) Avg. Ret (%)	(3) S.D. (%)	(4) SRatio	(5) CER (%)	(6) SRatio diff	(7) CER diff	(8) Ret-loss (%)	(9) Fee (bps)
Panel A: Prevailing mean benchmark								
HA	1.994	9.390	0.212	0.672				
Panel B: Option bounds								
Bound	0.007	15.790	0.000	−3.733	−0.212	−4.408	3.346	−442.558
Bound + Avg slackness	1.310	20.796	0.063	−5.177	−0.149	−5.841	3.106	−589.686
Bound (OLS)	2.701	18.569	0.145	−2.471	−0.067	−3.132	1.243	−316.664
Panel C: Time-series predictors								
$x_t = IS_{BW}$	5.651	20.548	0.275	−0.683	0.063	−1.348	−1.287	−140.101
$x_t = IS_{HJTZ}$	7.509	18.459	0.407	2.398	0.194*	1.732	−3.589	169.751
$x_t = SSI$	8.961	20.608	0.435	2.590	0.222*	1.946	−4.584	189.232
Panel D: Combined predictors								
$x_t = IS_{BW}$	7.503	19.281	0.389	1.926	0.177	1.260	−3.408	121.886
$x_t = IS_{HJTZ}$	8.403	19.470	0.432	2.717	0.219*	2.049	−4.268	200.599
$x_t = SSI$	10.154	19.342	0.525	4.543	0.313**	3.896	−6.047	385.479
Pooling	9.121	18.737	0.487	3.855	0.274**	3.194	−5.142	315.806

Table 5: R^2_{OS} statistics (in percent) for NBER-dated business cycles

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 1-month market excess return forecasts based on option bounds, time-series predictors, and combined predictors. We also report R^2_{OS} separately for NBER-dated expansions and recessions. The out-of-sample period is 2001:01–2020:12. In the last row of Panel C, we pool the individual forecasts from combined predictors by taking an arithmetic mean. Based on the Clark and West's (2007) test, * and ** indicate significance at the 10% and 5% levels for the positive R^2_{OS}, respectively.

Sub-periods	Overall	Expansions	Recessions
Panel A: Option bounds			
Bound	−1.450	0.768	−5.560
Bound + Avg slackness	−2.533	0.162	−7.527
Bound (OLS)	−8.445	−5.219	−14.425
Panel B: Time-series predictors			
$x_t = IS_{BW}$	−1.443	2.000**	−7.825
$x_t = IS_{HJTZ}$	−0.268	0.307*	−1.332
$x_t = SSI$	−0.776	−0.146	−1.944
Panel C: Combined predictors			
$x_t = IS_{BW}$	−0.788	2.321**	−6.549
$x_t = IS_{HJTZ}$	0.809**	0.814*	0.800
$x_t = SSI$	0.695**	−0.084	2.139
Pooling	1.417**	2.135**	0.088

Table 6: R^2_{OS} statistics (in percent) for longer-horizon forecasts

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 3-, 6-, and 12-month market excess return forecasts based on option bounds, stock market predictors, and combined predictors. In the last row of Panel C, we pool three individual forecasts from three combined predictors by taking an arithmetic mean. The out-of-sample period is 2001:01–2020:12. Based on the Clark and West's (2007) test, ** and *** indicate significance at the 5% and 1% levels for the positive R^2_{OS}, respectively.

Horizons	3-month	6-month	12-month
	Panel A: Option bounds		
Bound	0.799	5.582***	7.329***
Bound + Slackness	−2.693	−2.107	−13.936
Bound (OLS)	−7.572	8.250***	2.229**
	Panel B: Time-series predictors		
$x_t = IS_{BW}$	1.181***	6.227***	13.252***
$x_t = IS_{HJTZ}$	2.300***	5.861***	12.594***
$x_t = SSI$	3.471***	9.445***	12.671***
	Panel C: Combined predictors		
$x_t = IS_{BW}$	−1.136	2.341***	8.478***
$x_t = IS_{HJTZ}$	2.435***	6.421***	11.907***
$x_t = SSI$	4.055***	9.430***	15.711***
Pooling	6.065***	14.105***	26.975***

Table 7: R^2_{OS} statistics (in percent) for alternative function forms of $k(x_t)$

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 1-, 3-, 6-, and 12-month market excess return forecasts based on the combined predictors such that

$$b_t\,[k(x_t)] = k(x_t)\left[(T-t)R_{f,t}\text{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t},$$

where $R_{f,t}$ is the gross return on an risk-free asset, $\text{SVIX}^2_{t\to T}$ is computed from market prices of index options as in Martin (2017), and x_t is one of the sentiment variables. We consider two alternative forms for $k(x_t)$ as indicated by the panel headings. In the last row of each panel, we pool the individual forecasts from combined predictors by taking an arithmetic mean. The out-of-sample period is 2001:01–2020:12. Based on the Clark and West's (2007) test, *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively.

Horizons	1-month	3-month	6-month	12-month
	Panel A: Exponential form $k(x_t) = \exp(a+bx_t)+1$			
$x_t = IS_{BW}$	−0.879	−1.282	2.043***	8.041***
$x_t = IS_{HJTZ}$	0.883**	2.348***	5.885***	11.297***
$x_t = SSI$	0.768**	3.360***	8.528***	15.465***
Pooling	1.443**	5.783***	13.489***	26.456***
	Panel B: Linear form $k(x_t) = (a+bx_t)$			
$x_t = IS_{BW}$	−0.285	1.585***	6.087***	14.390***
$x_t = IS_{HJTZ}$	0.875**	2.690***	5.880***	13.904***
$x_t = SSI$	0.448**	3.575***	8.764***	14.387***
Pooling	1.325**	6.245***	13.746***	26.065***

Table 8: R^2_{OS} statistics (in percent) by using both SVIX and VIX

This table reports out-of-sample R^2 statistics (R^2_{OS}) in percent for 1-month market excess return based on the combined predictor. The combined predictor takes the form such that,

$$b_t\left[k(x_t)\right] = \begin{cases} k(x_t)\left[(T-t)R_{f,t}\text{SVIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}, & \text{if} \quad t \geq 1996 \\ k(x_t)\left[(T-t)R_{f,t}\text{VIX}^2_{t\to T} + R_{f,t}\right] - R_{f,t}, & \text{if} \quad t < 1996 \end{cases},$$

where $R_{f,t}$ is the gross return on an risk-free asset, $\text{SVIX}_{t\to T}$ is computed from market prices of index options as in Martin (2017), $\text{VIX}_{t\to T}$ is CBOE VIX index, x_t is one of three sentiment variables including Baker and Wurgler's (2006) sentiment index (IS_{BW}), Huang et al.'s (2015) sentiment index (IS_{HJTZ}) and Rapach et al.'s (2016) short interest index (SSI); and $k(x_t) = exp(a + bx_t)$. In the last row of Panel C, we pool the individual forecasts from combined predictors by taking an arithmetic mean. The full sample spans from 1990 to 2020, and we consider four out-of-sample testing periods as indicated by the column headings below. Based on the Clark and West's (2007) test, *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively.

Out-of-sample periods	1996−2020	2001−2020	2006−2020	2011−2020
Panel A: Option bounds by SVIX and VIX				
Bound	−0.775	−1.45	−2.214	0.741
Bound + Slackness	−0.683	−2.024	−2.268	2.436
Bound (OLS)	−4.108	−6.761	−8.018	−2.277
Panel B: Time-series predictors				
$x_t = IS_{BW}$	−0.138	−0.513	−0.128	1.979*
$x_t = IS_{HJTZ}$	0.217*	0.730*	0.525	0.87
$x_t = SSI$	0.822**	0.303**	1.798**	0.749*
Panel C: Combined predictors				
$x_t = IS_{BW}$	−0.187	−0.061	−0.176	1.822*
$x_t = IS_{HJTZ}$	−0.937	1.625**	−0.103	1.528*
$x_t = SSI$	1.027**	1.498**	2.027**	1.279*
Pooling	0.782**	1.686**	0.825*	1.872*

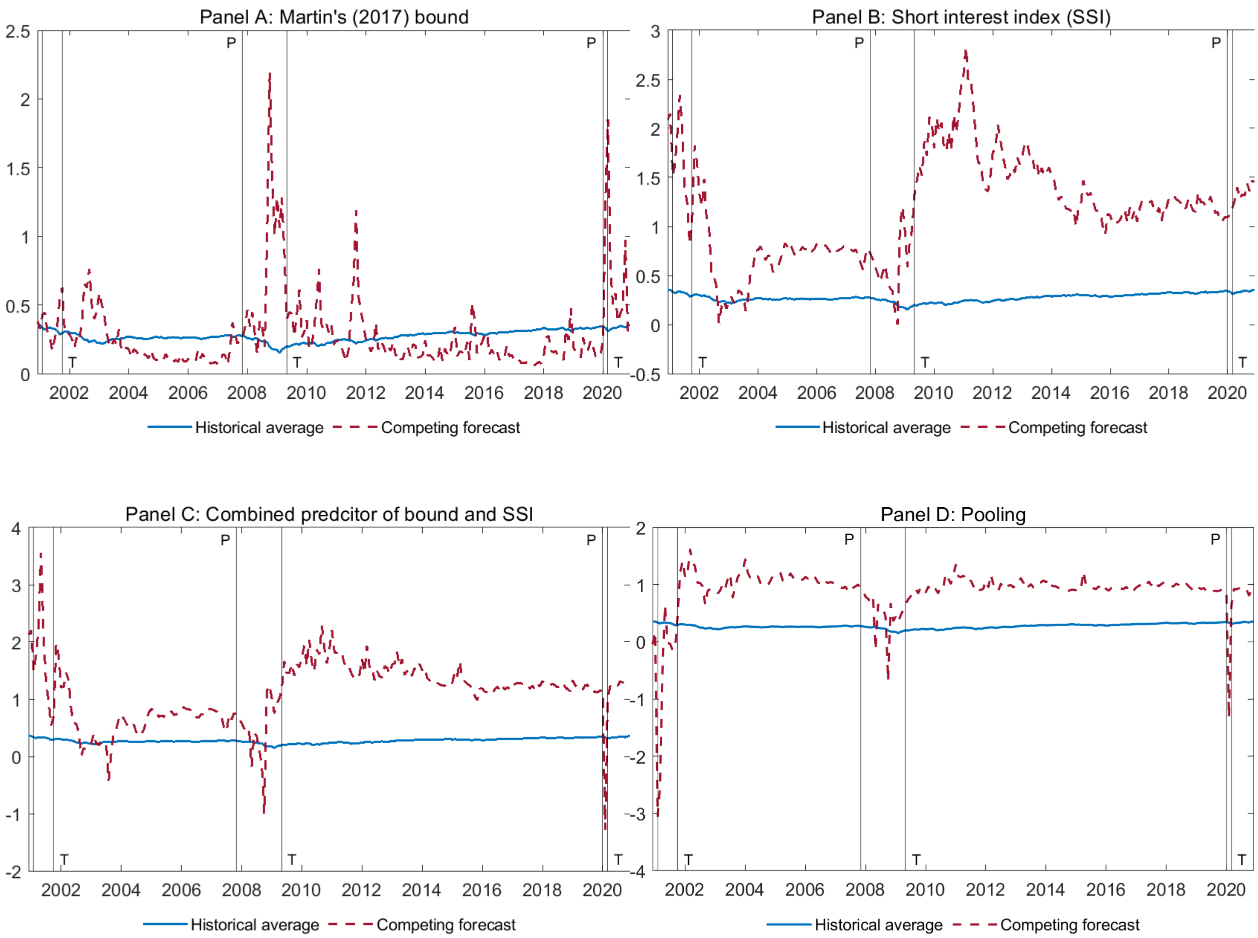

Figure 1: Out-of-sample forecasts of market excess return

This figure depicts out-of-sample market excess return forecasts (in percent) for 2001:01–2020:12 based on the predictor (or method) in the panel heading, along with the historical average benchmark. Pooling stands for aggregating the three individual forecasts from three combined predictors with an arithmetic mean. The vertical lines delineate NBER-dated business-cycle recessions/expansions.

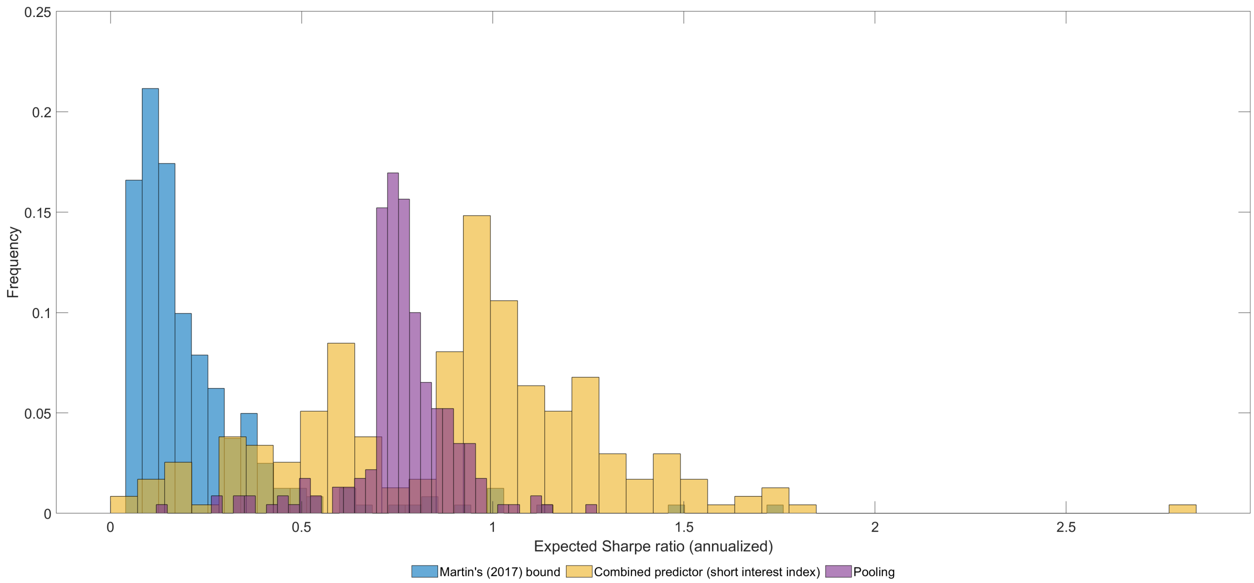

Figure 2: Distribution of ex-ante Sharpe ratios

This figure plots the *ex-ante* Sharpe ratio perceived by the investor at time t when forecasting the return in next period for 2001:01–2020:12,

$$SR_{j,t} = \frac{\hat{r}_{j,t+1}}{\hat{\sigma}_{t+1}},$$

where $\hat{r}_{j,t+1}$ is the forecasted excess return on S&P 500 index at time t based on either Martin (2017), combining bound and short interest index, or pooling across three individual forecasts from combined predictors, and $\hat{\sigma}_t$ is the forecasted volatility computed from historical return using a 5-year rolling window.

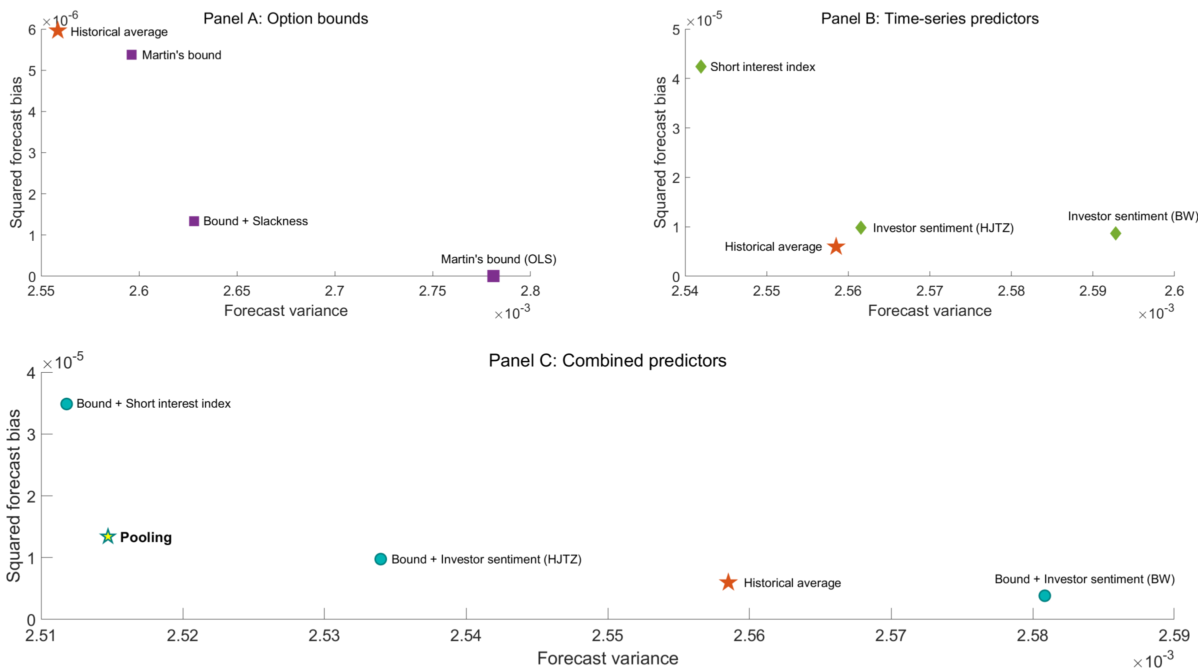

Figure 3: Bias-Variance decomposition

This figure plots the bias-variance decomposition of the MSPE based on the forecasts from option bounds, stock market predictors, and combined predictors in the out-of-sample period 2001:01–2020:12.

Figure 4: Log cumulative excess returns for portfolios out-of-sample

Each panel depicts the log cumulative excess return for a portfolio constructed using the market excess return forecast in the panel heading and the historical mean benchmark forecast for the period 2001–2020. Vertical lines indicate NBER-dated business-cycle peaks (P) and troughs (T).

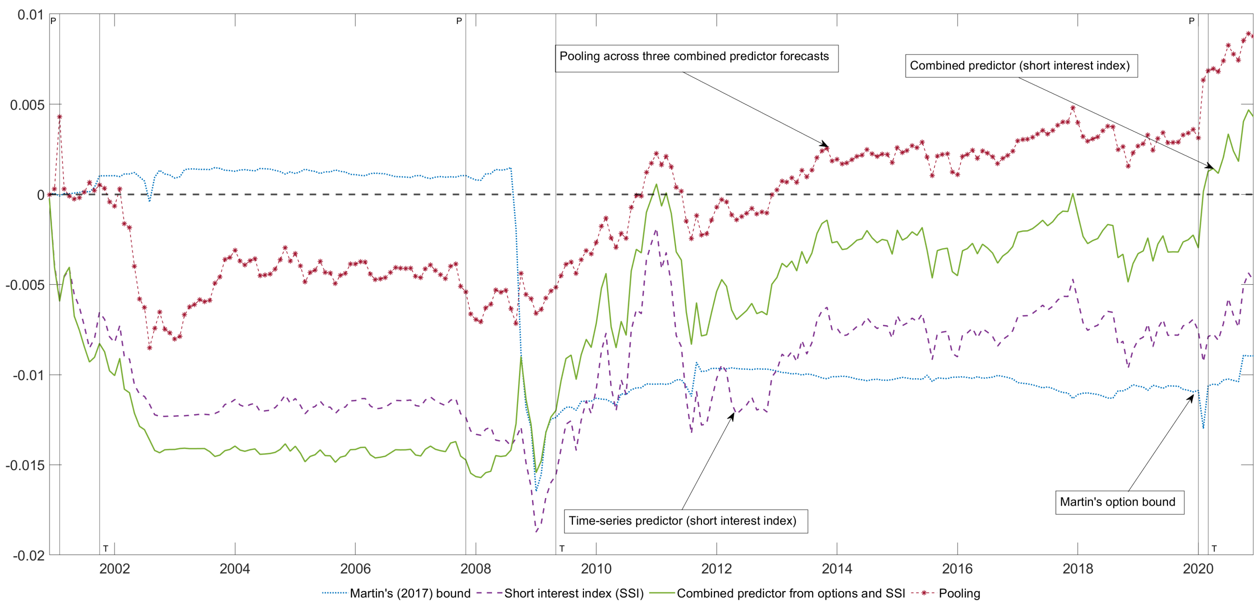

Figure 5: Cumulative error difference relative to historical average

This figure plots the cumulative square prediction error for 2001:01–2020:12 such that

$$\text{square error difference} = (r_t - \bar{r}^{HA}_{t|t-1})^2 - (r_t - \hat{r}_{t|t-1})^2,$$

where $\bar{r}^{HA}_{t|t-1}$ is the historical average (HA), r_t is the realized market excess return, and $\hat{r}_{|t-1}$ is the forecast based on an option bound, a time-series (stock market) predictor, or a combined predictor. Pooling stands for aggregate the three individual forecasts from three combined predictors.

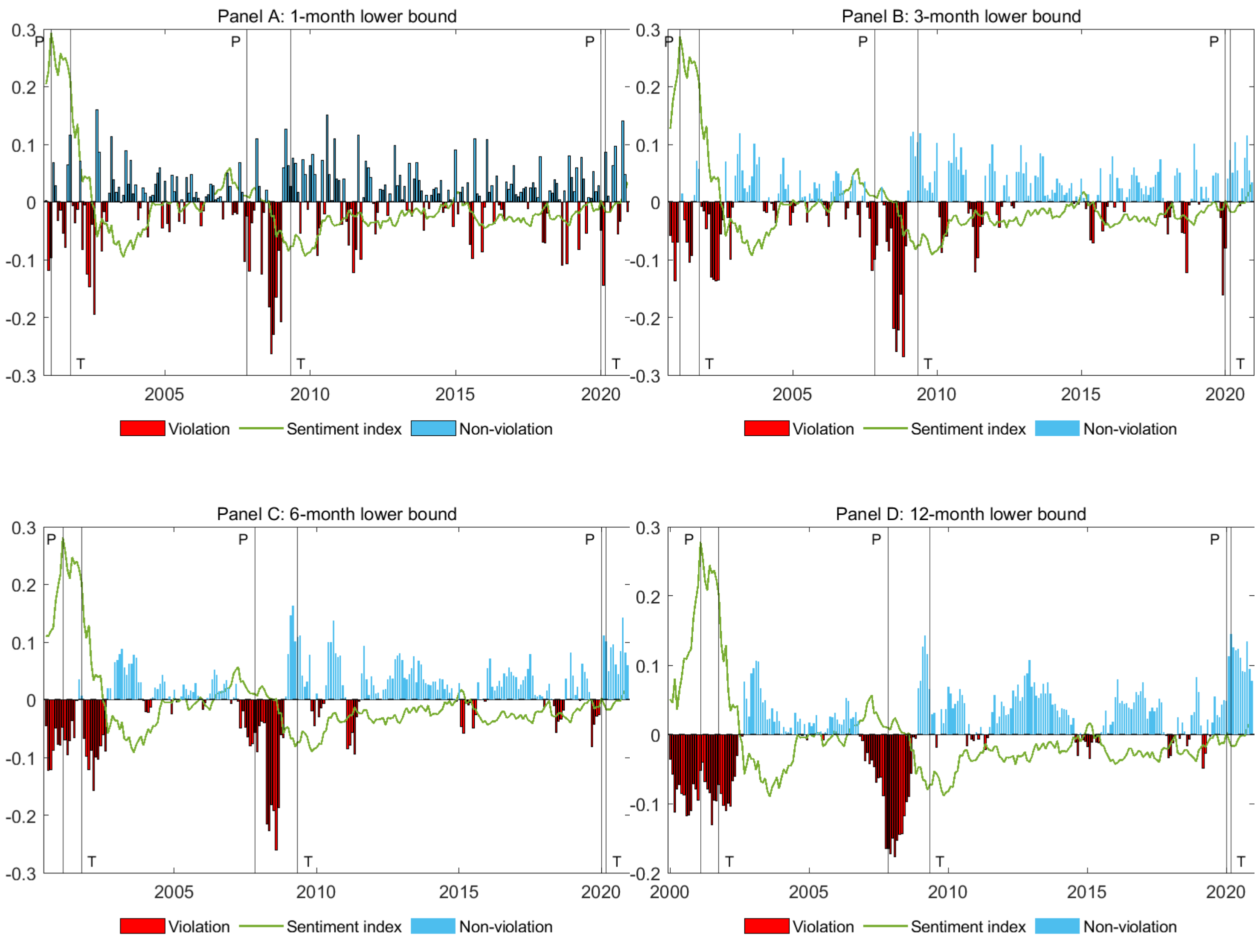

Figure 6: Violation of the lower bound by Martin (2017)

This figure plots the violation of the inequality in Martin (2017) for 2001:01–2020:12 such that

$$\mathbb{E}_t(R_T) - R_{f,t} \geq \frac{1}{R_{f,t}} Var_t^{\mathbb{Q}}(R_T).$$

The red (blue) bar corresponds to the violation (non-violation) of the above inequality, and the green line plots the investor sentiment by Baker and Wurgler (2006), which is normalized to 1 unit.

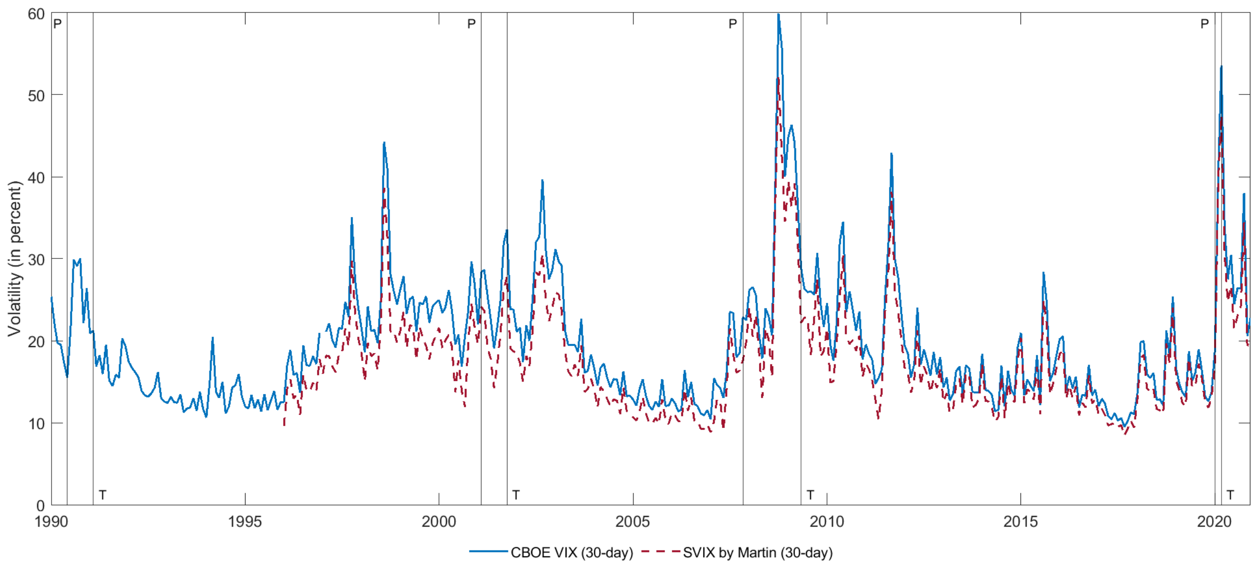

Figure 7: CBOE VIX and SVIX (in percent)

This figure plots CBOE VIX index and Martin's (2017) SVIX index (both in percent) for the period 1990:01–2020:12.